Exercise 16

Graph the two basic solutions along with several other solutions of the differential equation. What features do the solutions have in common?

$$2\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \to \quad \frac{dy}{dx} = re^{rx} \quad \to \quad \frac{d^2y}{dx^2} = r^2e^{rx}$$

Plug these formulas into the ODE.

$$2(r^2e^{rx}) + re^{rx} - e^{rx} = 0$$

Divide both sides by e^{rx}.

$$2r^2 + r - 1 = 0$$

Solve for r.

$$r = \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)} = \frac{-1 \pm \sqrt{9}}{4} = -\frac{1}{4} \pm \frac{3}{4}$$

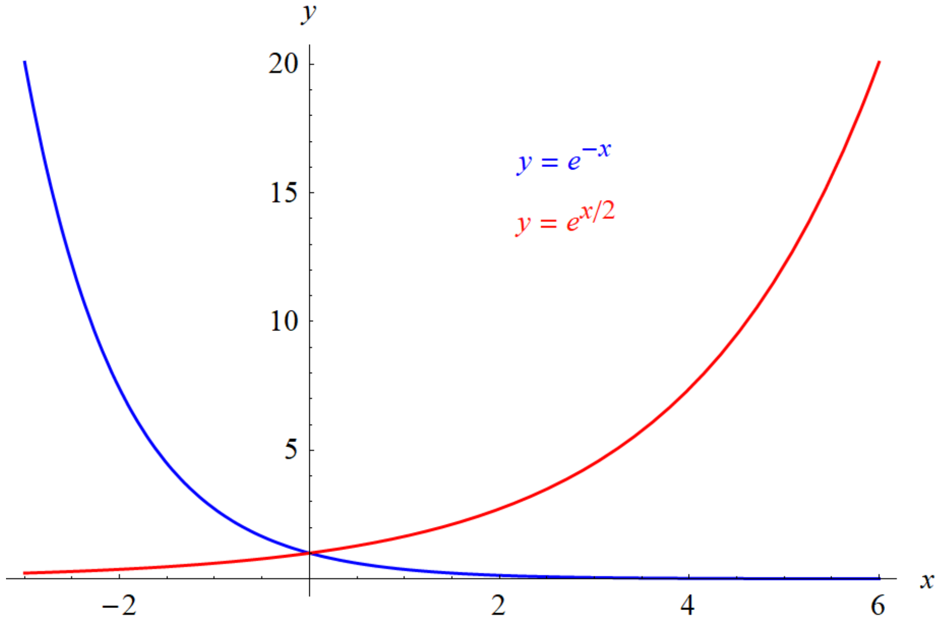
$$r = \left\{-1, \frac{1}{2}\right\}$$

Two solutions to the ODE are e^{-x} and $e^{x/2}$. By the principle of superposition, then,

$$y(x) = C_1e^{-x} + C_2e^{x/2},$$

where C_1 and C_2 are arbitrary constants.

Below is a graph of these two solutions.



e^{-x} blows up as $x \to -\infty$, and $e^{x/2}$ blows up as $x \to \infty$.